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Semiclassical quantization of the giant magnon

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Abstract: Solitons in field theory provide a window into regimes not directly accessible by the fundamental perturbative degrees of freedom. Motivated by interest in the worldsheet S-matrix of string theory in $AdS_5 \times S^5$ in the limit of infinite worldsheet volume we consider the semiclassical quantization of a particular soliton of this theory: the Hofman-Maldacena 'giant magnon' spinning string. We obtain explicit formulas for the complete spectrum of bosonic and fermionic fluctuations around the giant magnon. As an application of these results we confirm that the one-loop correction to the classical energy vanishes as expected.

Keywords: Solitons Monopoles and Instantons, AdS-CFT Correspondence, Integrable Equations in Physics.

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Contents

1. Introduction

It is difficult to overstate the important role that solitons play in quantum field theory, especially in integrable theories. Because of their particle-like nature their dynamics can be effectively captured by a small (in particular, finite) number of collective degrees of freedom. Quantization of these collective coordinates following the work of Gervais, Jevicki and Sakita [1 − 4] provides a systematic framework for studying the theory in regimes not directly accessible by perturbation theory involving the original degrees of freedom.

An interesting soliton to emerge in recent studies of the AdS/CFT correspondence is the giant magnon of Hofman and Maldacena [5]. This is a soliton of the integrable [6, 7] $AdS_5 \times S^5$ worldsheet sigma-model [8] (actually living inside an $\mathbb{R} \times S^2$ subspace) whose image in spacetime is a stretched string that is pointlike in AdS_5 and rotates uniformly around an axis of an $S^2 \subset S^5$. Its name derives from the fact that it is the dual description of an elementary excitation (magnon) in the spin chain description [9 − 15] of the dual $\mathcal{N} = 4$ gauge theory.

String theory on $\mathbb{R} \times S^2$ is classically equivalent to sine-Gordon theory [16] and the giant magnon is the image of the sine-Gordon soliton under this equivalence. Classical aspects of the giant magnon, such as the phase shift for magnon scattering [5], can be understood in this way. However string theory on $AdS_5 \times S^5$ is much more than just the reduced $\mathbb{R} \times S^2$ sigma-model, having additional bosonic, fermionic and (in conformal gauge) ghost degrees of freedom. Correspondingly the giant magnon has a richer set of collective coordinates ($\mathbb{R} \times S^3$, together with eight fermionic zero modes [17] which upon quantization lead to 16 polarization states) than the sine-Gordon soliton (just the soliton's

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position $\mathbb{R}$). The dressing method [18] (or the Bäcklund transformation [19]) can be used to construct classical solutions [20, 21] describing arbitrary scattering and bound states (both BPS [22, 23] and non-BPS [5]) of magnons. Some of the latter have played an important role recently [24] in elucidating the nature of double poles [25, 26] in the magnon S-matrix [15, 27 – 29]. Other related aspects of giant magnons and the S-matrix have been studied recently in [30 – 42].

The study of classical spinning strings [43] and especially their semiclassical quantization (see in particular [44, 45] and the many additional references given below) has been incredibly fertile ground for quantitative studies of AdS/CFT. Typically the semiclassical analysis is done at the level of the lagrangian, by expanding the action to quadratic order in small fluctuations around the classical background and looking for appropriate field redefinitions to decouple various independent modes from each other. One aspect of the giant magnon that sets it apart from much of this extensive literature is the fact that it is not at rest on the worldsheet (in conformal gauge, which is by far the most natural gauge to use in this case). This fact makes it more natural to carry out the semiclassical analysis not at the level of fields in the lagrangian but rather by working out an explicit basis of eigenmodes (i.e., asymptotically plane-wave solutions of the linearized equations of motion around the soliton background). The present work was motivated in part by [17] where Minahan explicitly constructed the fermionic zero modes of the giant magnon but left analysis of the non-zero mode fluctuations tantalizingly open in his equation (4.1). Our work benefits in particular from using the same convenient form of κ-symmetry fixing employed in [17].

The outline of this paper is as follows. In section 2 we review the giant magnon solution and display explicit solutions for small bosonic fluctuations around the classical background. This part requires relatively little work since the dressing method can be used to algebraically construct a complete set of small fluctuations around any N-soliton configuration. We do not know of a similar construction for fermionic fluctuations, which we therefore carry out 'by hand' in section 3. In fact the fermionic analysis turns out to be much simpler than we had any right to expect. Some steps of the calculation fall together in an almost miraculous way, encouraging us to speculate that properly understanding the structure should make it feasible to construct fermionic fluctuations around more complicated multi-soliton backgrounds.

Finally in section 4 we use our results to read off the stability angles which appear in the evaluation of the 1-loop functional determinant in the soliton background, following the method pioneered by Dashen, Hasslacher and Neveu [46]. We show that the first $\mathcal{O}(1/\sqrt{\lambda})$ correction to the classical 'energy' $\Delta - J$ of the giant magnon vanishes. Although this particular result is in accord with the expectation based on the $\mathrm{SU}(2|2) \ltimes \mathbb{R}^2$ superalgebra [15, 5], continued study of quantized giant magnons should provide an approach to probing the $AdS_5 \times S^5$ worldsheet S-matrix that complements studying it via the scattering of the fundamental worldsheet degrees of freedom [47]. The latter approach has proven to be surprisingly successful at one loop [48] in a certain limit introduced by Maldacena and Swanson [49]. Unfortunately taking this limit seems to complicate the analysis of giant magnons considerably, even at the classical level.

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2. Bosonic sector

We write the bosonic part of the $AdS_5 \times S^5$ sigma-model in the form [50]

$$S = -\frac{\sqrt{\lambda}}{2\pi} \int dt\, dx\, \left[\eta^{ab}\partial_a Y^\mu \partial_b Y_\mu + \widetilde{\Lambda}(Y^2+1)\right] + \left[\eta^{ab}\partial_a X^i \partial_b X_i + \Lambda(X^2-1)\right], \qquad (2.1)$$

with $\eta^{ab} = \mathrm{diag}(-1,+1)$ and λ identified with the 't Hooft coupling in the dual gauge theory. Here Y^μ are coordinates on $\mathbb{R}^{4,2}$, X^i are coordinates on $\mathbb{R}^6$, and the Lagrange multipliers $\widetilde{\Lambda}$ and Λ enforce the embedding constraints

$$Y^2 = -1\,, \qquad X^2 = +1 \qquad (2.2)$$

onto AdS_5 and S^5 respectively. The sigma-model equations of motion

$$(\partial^2 - \widetilde{\Lambda})Y = (\partial^2 - \Lambda)X = 0 \qquad (2.3)$$

are to be supplemented in string theory by the Virasoro constraints

$$\begin{aligned} T_{00} &= \frac{1}{2}\left[(\partial_t Y)^2 + (\partial_x Y)^2 + (\partial_t X)^2 + (\partial_x X)^2\right] = 0\,, \\ T_{01} &= \partial_t Y^\mu \partial_x Y_\mu + \partial_t X^i \partial_x X_i = 0\,. \end{aligned} \qquad (2.4)$$

From (2.2) and (2.3) it follows that the classical values of the Lagrange multipliers are

$$\widetilde{\Lambda} = -Y^\mu \partial^2 Y_\mu\,, \qquad \Lambda = +X^i \partial^2 X_i\,. \qquad (2.5)$$

2.1 The giant magnon

For the rest of the paper we will be interested in a particular classical solution of the equations (2.3) and (2.4), namely the giant magnon of [5]. This is an open string which is pointlike in AdS_5 and extended along the S^5, with endpoints moving at the speed of light along an equator of the S^5, which we choose to lie in the X^5–X^6 plane. We introduce the notation $\vec{X}$ to refer to the four transverse coordinates $X^1, \ldots, X^4$. A physical closed string is obtained by attaching two or more giant magnons to each other end to end. For our purposes we can ignore this step because the individual pieces of string do not talk to each other.

The giant magnon is characterized by a 'momentum' $p \in [0, 2\pi)$ and by the choice of a point on S^3, i.e. by a a four-component unit vector $\vec{n}$, which specifies the polarization of the magnon in the transverse directions. Choosing Y^0 and Y^5 to be the two timelike directions in $\mathbb{R}^{4,2}$, the giant magnon solution in our coordinates takes the form

$$\begin{aligned} Y^0 + iY^5 &= e^{it}\,, \\ \vec{X} &= \vec{n}\, \sin\tfrac{p}{2}\, \mathrm{sech}\, u\,, \\ Z \equiv X^5 + iX^6 &= e^{it}\left[\cos\tfrac{p}{2} + i \sin\tfrac{p}{2} \tanh u\right], \end{aligned} \qquad (2.6)$$

where

$$u = (x - t\cos\tfrac{p}{2}) \csc\tfrac{p}{2}\,. \qquad (2.7)$$

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The Lagrange multipliers (2.5) are classically equal to

$$\widetilde{\Lambda} = 1\,, \qquad \Lambda = 1 - 2\,\mathrm{sech}^2\, u\,. \tag{2.8}$$

It is evident that the solution (2.6) describes a localized wave (i.e., a soliton) polarized in the $\vec{n}$ direction which travels along the worldsheet with velocity $\cos\frac{p}{2}$. The quantity p is not a Noether charge of the action (2.3) (the charge associated with x translations is identically zero due to the Virasoro constraints). Rather it is a parameter of the soliton (2.6) which specifies the difference in longitude between the two endpoints of the string. The quantity $e^{ip} - 1$ appears as a central charge in the supersymmetry algebra of the giant magnon [15, 5, 37], and while individual magnons can have arbitrary p, physical closed string states have total momentum $p \in 2\pi\mathbb{Z}$ so that $e^{ip} - 1 = 0$.

The giant magnon has four bosonic collective coordinates (zero modes) parameterizing $\mathbb{R} \times S^3$. The S^3 coordinate is just the polarization vector $\vec{n}$ we already encountered. The $\mathbb{R}$ coordinate, which we may call q, denotes the position of the magnon on the worldsheet; this collective coordinate may be introduced by replacing $x \to x - q$ in (2.7). In addition the giant magnon is a BPS state and there are eight fermionic collective coordinates which have been explicitly constructed in [17].

2.2 AdS_5 fluctuation spectrum

We now turn to the analysis of the spectrum of fluctuations around the giant magnon solution (2.6), beginning with the AdS_5 bosons, where we find one massless and four massive fluctuations. The reader is unlikely to be surprised by this result, but it is instructive to treat the analysis carefully in order to set the stage for the following sections.

The equation of motion for a fluctuation δY^μ around the giant magnon solution, after eliminating the Lagrange multiplier, is

$$(\partial^2 - 1)\delta Y^\mu + Y^\mu Y_\nu \partial^2 \delta Y^\nu = 0\,, \tag{2.9}$$

subject to the constraint

$$Y_\mu\, \delta Y^\mu = 0\,. \tag{2.10}$$

The massless fluctuation is exhibited by solving (2.10) with the ansatz

$$\delta Y^0 = -f \sin t\,, \qquad \delta Y^5 = +f \cos t\,. \tag{2.11}$$

Substituting this into (2.9) we find that f satisfies the free wave equation

$$\partial^2 f = 0\,. \tag{2.12}$$

In a proper quantization of string theory on $AdS_5 \times S^5$ the sigma-model action (2.1) would be supplemented by ghosts which would cancel the mode (2.12) together with a similar S^5 mode that we will find below. These are analogous to longitudinal fluctuations in light-cone gauge, and it is sufficient for the purpose of our semiclassical analysis to simply omit them [44, 51].

The remaining four fluctuations $\delta\vec{Y}$ in the transverse spatial directions of AdS_5 satisfy (2.10) automatically, and moreover for these modes the second term in (2.9) vanishes, giving four free bosons with mass $m^2 = 1$. Note that these modes preserve the Virasoro constraints (2.4) to first order in the fluctuation.

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2.3 S^5 fluctuation spectrum

The equation of motion for an S^5 fluctuation δX^i around (2.6) is

$$(\partial^2 - 1 + 2\,\mathrm{sech}^2\, u)\delta X^i - X^i X_j \partial^2 \delta X^j = 0 \tag{2.13}$$

subject to

$$X_i\, \delta X^i = 0\,. \tag{2.14}$$

These equations admit two different classes of solutions.

The first class of solutions are the zero modes which have been discussed in [17] and are in one-to-one correspondence with the collective coordinates. The choice of polarization $\vec{n}$ breaks the SO(4) symmetry of the transverse directions, leading to the three independent zero modes

$$\delta\vec{X} = \vec{m}\,\mathrm{sech}\, u\,, \qquad \vec{m}\cdot\vec{n} = 0\,. \tag{2.15}$$

Moreover the presence of the magnon breaks the x-translation symmetry, leading to the zero mode

$$\begin{aligned} \delta\vec{X} &= -\vec{n}\,\mathrm{sech}\, u \tanh u\,, \\ \delta Z &= i e^{it}\,\mathrm{sech}^2\, u\,. \end{aligned} \tag{2.16}$$

These particular solutions will not play any further role in our analysis.

Instead of these normalizable zero modes we are interested in plane wave fluctuations of the form

$$e^{iku - i\omega v} f(u)\,, \tag{2.17}$$

where $f(u)$ is some profile and we have introduced

$$v = (t - x\cos\tfrac{p}{2})\csc\tfrac{p}{2} \tag{2.18}$$

as the time variable in the magnon's 'rest frame' complementing u as defined in (2.7). The variables u and v resemble variables appropriate for a relativistic system boosted to velocity $\cos\frac{p}{2}$, but it should be kept in mind that the giant magnon solution (2.6) is not relativistically invariant [5]. In particular, recall that the parameter p represents the physical separation in longitude of the two endpoints of the string on S^2. The appearance of these apparently boosted variables emerges from the fact that string theory on $\mathbb{R}\times S^2$ is classically equivalent to relativistic sine-Gordon theory.

One of the plane-wave solutions of (2.13) is massless. A basis for the positive energy excitations of this mode is given by

$$\begin{aligned} \delta\vec{X} &= e^{iku - i|k|v}\vec{n}\,(k + |k|\cos\tfrac{p}{2})\,\mathrm{sech}\, u \tanh u\,, \\ \delta Z &= -i e^{iku - i|k|v} e^{+it}\left[k - |k|\sinh u \sinh(u + i\tfrac{p}{2})\right]\mathrm{sech}^2\, u\,, \\ \delta\overline{Z} &= +i e^{iku - i|k|v} e^{-it}\left[k - |k|\sinh u \sinh(u - i\tfrac{p}{2})\right]\mathrm{sech}^2\, u\,, \end{aligned} \tag{2.19}$$

where $k \in \mathbb{R}$ is the wavenumber. In string theory we will omit this mode as discussed beneath (2.12).

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The remaining four physical fluctuations are characterized by a polarization vector $\vec{m}$ in the transverse $\mathbb{R}^4$. The corresponding positive frequency excitations are

$$\begin{aligned}
\delta\vec{X} &= e^{iku-i\omega v}\left[\vec{m}(k+i\tanh u) - \vec{n}(n\cdot m)\left(k+\omega\cos\tfrac{p}{2}\right)\operatorname{sech}^2 u\right], \\
\delta Z &= -ie^{iku-i\omega v}e^{+it}(n\cdot m)\left[k\sinh u + \omega\sinh(u+i\tfrac{p}{2}) + i\cosh u\right]\operatorname{sech}^2 u\,, \\
\delta\overline{Z} &= +ie^{iku-i\omega v}e^{-it}(n\cdot m)\left[k\sinh u + \omega\sinh(u-i\tfrac{p}{2}) + i\cosh u\right]\operatorname{sech}^2 u\,,
\end{aligned} \tag{2.20}$$

where

$$\omega = +\sqrt{k^2+1} \tag{2.21}$$

indicating that the effective mass of these modes is $m^2 = 1$. Indeed, very far from the soliton core ($|u| \gg 1$) the fluctuation $\delta\vec{X}$ satisfies the free massive wave equation and its profile becomes constant. Note that the three fluctuations orthogonal to the original giant magnon ($n \cdot m = 0$) take the very simple form

$$\delta\vec{X} = e^{iku-i\omega v}\vec{m}(k+i\tanh u)\,. \tag{2.22}$$

Although it is straightforward to verify that (2.20) satisfies (2.13) and (2.14) as claimed, a comment is in order regarding how we obtained this rather complicated formula. In fact it is well known [46] (see [4, 52] for further examples) that a complete basis of bosonic fluctuations around any N-soliton solution in a classically integrable theory can be constructed algebraically, for example by the dressing method [18] or equivalently the Bäcklund transformation (see in particular [19] for a nice discussion in this context). One begins with the desired background solution and then superposes on top of it a soliton-antisoliton bound state (i.e., a breather). Expanding the resulting solution to first order in the (complex) momentum of the breather yields the desired fluctuation of the background solution. Unfortunately we know of no similarly simple way to determine the fermionic fluctuations in the following section.

3. Fermionic sector

In this section we obtain explicit formulas for the complete spectrum of fermionic fluctuations around the giant magnon solution (2.6). The classical background has $\vartheta = 0$ so we can use the variable ϑ itself to denote the fluctuation, rather than the more cumbersome alternative $\delta\vartheta$. The action for the fermionic fluctuations around a general classical string solution $\mathrm{X}^\mu(t,x)$ is [8]

$$S = \frac{\sqrt{\lambda}}{2\pi}\int dt\,dx\,\left[i(\eta^{ab}\delta^{IJ} - \epsilon^{ab}s^{IJ})\bar{\vartheta}^I\rho_a D_b\vartheta^J + \mathcal{O}(\vartheta^4)\right]. \tag{3.1}$$

The quartic terms will play no role in our linearized analysis of fluctuations around $\vartheta = 0$. Here ϑ^I are two 10-dimensional Majorana-Weyl spinors, $s^{IJ} = \mathrm{diag}(+1,-1)$,

$$\rho_a = \Gamma_A E^A_\mu(\mathrm{X})\partial_a \mathrm{X}^\mu \tag{3.2}$$

are projections of the ten-dimensional Dirac matrices involving the $AdS_5 \times S^5$ vielbein E^A_μ and the bosonic coordinates X^μ of the classical solution. Note that μ is now a curved space index denoting all of the $AdS_5 \times S^5$ directions, rather than the flat space index in the embedding space $\mathbb{R}^{4,2}$ of the AdS_5 part only, that we used in the previous section. The covariant derivative appearing in (3.1) is given by

$$D_a\vartheta^I = \left(\delta^{IJ}\left(\partial_a + \frac{1}{4}\omega_\mu^{AB}\partial_a \mathrm{X}^\mu \Gamma_{AB}\right) - \frac{i}{2}\epsilon^{IJ}\Gamma_*\rho_a\right)\vartheta^J \tag{3.3}$$

in terms of the usual spin connection ω_μ^{AB} and the matrix

$$\Gamma_* = i\Gamma_{01234} \tag{3.4}$$

satisfying $\Gamma_*^2 = 1$. Whenever necessary, we use for concreteness a Majorana representation of imaginary Dirac matrices with Γ_0 symmetric and the other nine matrices antisymmetric.

3.1 The fluctuation equations

The linearized equations of motion following from (3.1) are

$$\begin{aligned}(\rho_0 - \rho_1)(D_0 + D_1)\vartheta^1 &= 0\,,\\ (\rho_0 + \rho_1)(D_0 - D_1)\vartheta^2 &= 0\,.\end{aligned} \tag{3.5}$$

The full Green-Schwarz action for $AdS_5 \times S^5$, including both (2.1) and (3.1), possesses a local fermionic symmetry (κ-symmetry) under which the physical solutions must be gauge-fixed. The zero mode solutions of (3.5) were recently constructed by Minahan [17], who noted that it is simplest to first find solutions without worrying about κ-symmetry and impose a projection only at the end of the calculation. We will find that the non-zero mode solutions of (3.5) enjoy this simple κ-fixing as well.

We begin by following [17] to process (3.5) into a more convenient form. We change variables from (x, t) to the coordinates

$$\begin{aligned}u &= (x - t\cos\tfrac{p}{2})\csc\tfrac{p}{2}\,,\\ v &= (t - x\cos\tfrac{p}{2})\csc\tfrac{p}{2}\end{aligned} \tag{3.6}$$

introduced in (2.7) and (2.18) above (note that these were denoted respectively by (x, ξ) in [17]). The equations for ϑ^I can be decoupled and written as

$$i(\rho_0 - \rho_1)\left[\frac{1}{\tanh u}(\mathcal{D} - \partial_v)\frac{1}{\tanh u}(\mathcal{D} + \partial_v)\vartheta^1 - \vartheta^1\right] = 0\,, \tag{3.7}$$

$$i(\rho_0 + \rho_1)\left[\frac{1}{\tanh u}(\widetilde{\mathcal{D}} - \partial_v)\frac{1}{\tanh u}(\widetilde{\mathcal{D}} + \partial_v)\vartheta^2 - \vartheta^2\right] = 0\,, \tag{3.8}$$

where

$$\begin{aligned}\mathcal{D} &= \partial_u + \frac{1}{2}G\Gamma_{\phi\theta}, &\quad G &= +\frac{1}{\cosh u}\left(1 + \frac{\cos\frac{p}{2}}{\tanh^2 u + \cos^2\frac{p}{2}\,\mathrm{sech}^2\, u}\right),\\ \widetilde{\mathcal{D}} &= \partial_u + \frac{1}{2}\widetilde{G}\Gamma_{\phi\theta}, &\quad \widetilde{G} &= -\frac{1}{\cosh u}\left(1 - \frac{\cos\frac{p}{2}}{\tanh^2 u + \cos^2\frac{p}{2}\,\mathrm{sech}^2\, u}\right).\end{aligned} \tag{3.9}$$

The subscripts on $\Gamma_{\phi\theta}$ refer to the usual angular coordinates (θ, ϕ) parameterizing the $S^2 \subset S^5$ on which the giant magnon (2.6) lives (i.e. $\vec{X} = \vec{n}\cos\theta$, $Z = e^{i\phi}\sin\theta$). The matrix $\Gamma_{\phi\theta}$ satisfies $(\Gamma_{\phi\theta})^2 = -1$, and in fact this is all we need to know about it.

The matrices $\rho_0 \pm \rho_1$ each have half the maximal rank and commute with the differential operators $\mathcal{D}$ and $\widetilde{\mathcal{D}}$. Following [17], we can therefore first solve (3.7) and (3.8) for ϑ^I and then treat the projected spinors

$$\Psi^1 = i(\rho_0 - \rho_1)\vartheta^1\,, \qquad \Psi^2 = i(\rho_0 + \rho_1)\vartheta^2 \tag{3.10}$$

as the fields fixed under κ-symmetry. For later reference we note equation (2.16) from [17], which in our notation reads

$$\tan\tfrac{p}{4}(\mathcal{D} + \partial_v)\Psi^1 - \tfrac{i}{2}\Gamma_*(\bar{\rho}_0 - \rho_0)\Psi^2 = 0\,, \tag{3.11}$$

where $\bar{\rho}_0 = -\rho_0^\dagger = \Gamma_*\rho_0\Gamma_*$. Since the matrix $(\bar{\rho}_0 - \rho_0)$ is nonsingular this equation determines Ψ^2 completely once Ψ^1 is known.

3.2 Solving the equation for ϑ

Here we present the technical details involved in solving the equations (3.7) and (3.8). The reader interested only in the final answers may turn to the next subsection. Since the two equations have nearly identical form we begin by focusing on the unprojected version of the first equation

$$\frac{1}{\tanh u}(\mathcal{D} - \partial_v)\frac{1}{\tanh u}(\mathcal{D} + \partial_v)\vartheta^1 - \vartheta^1 = 0\,. \tag{3.12}$$

We proceed by transforming the second order equation (3.12) into a system of two coupled first ordered equations

$$\begin{aligned} \frac{1}{\tanh u}\,(\mathcal{D} + \partial_v)\,\vartheta^1 &= \widetilde{\vartheta}^1\,, \\ \frac{1}{\tanh u}\,(\mathcal{D} - \partial_v)\,\widetilde{\vartheta}^1 &= \vartheta^1 \end{aligned} \tag{3.13}$$

involving ϑ^1 and a new field $\widetilde{\vartheta}^1$ (which in fact is clearly related to ϑ^2 thanks to (3.11)). In what follows we employ the matrix notation

$$\Theta = \begin{pmatrix} \vartheta^1 \\ \widetilde{\vartheta}^1 \end{pmatrix}. \tag{3.14}$$

The zero mode solutions satisfying $\partial_v\vartheta^I = 0$ were constructed by Minahan in [17]. For the non-zero modes we make a Fourier ansatz for the v dependence

$$\Theta(v, u) = e^{-i\omega v}\Theta(u)\,. \tag{3.15}$$

Since $(\Gamma_{\phi\theta})^2 = -1$ we can decompose Θ into eigenspinors as

$$\Theta = \Theta_+ + \Theta_-, \qquad \Gamma_{\phi\theta}\Theta_\pm = \pm i\Theta_\pm\,. \tag{3.16}$$

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Then using $\mathcal{D} = \partial_u + \frac{1}{2} G\Gamma_{\phi\theta}$ we see that (3.13) can be rearranged into the matrix equation

$$(\partial_u - A_\pm)\Theta_\pm = 0, \qquad A_\pm = \begin{pmatrix} i(\omega \mp \frac{G}{2}) & \tanh u \\ \tanh u & i(\omega \pm \frac{G}{2}) \end{pmatrix}. \tag{3.17}$$

Our strategy is to find an invertible transformation

$$\Theta_\pm \to \Theta'_\pm = S\Theta_\pm \tag{3.18}$$

that diagonalizes (3.17), solve the system of equations for $\Theta'_\pm$ and then transform back to find the result for $\Theta_\pm$.

In the transformed variables (3.17) becomes

$$\partial_u \Theta'_\pm = H_\pm \Theta'_\pm, \qquad H_\pm = (\partial_u S + S A_\pm) S^{-1}\,. \tag{3.19}$$

Parameterizing

$$S = \begin{pmatrix} a(u) & b(u) \\ c(u) & d(u) \end{pmatrix} \tag{3.20}$$

we find that $H_\pm$ are both diagonal if these entries satisfy

$$\begin{aligned} a\,b' - b\,a' + (a^2 - b^2)\tanh u - 2i\omega ab &= 0\,, \\ c\,d' - d\,c' + (c^2 - d^2)\tanh u - 2i\omega cd &= 0\,, \end{aligned} \tag{3.21}$$

where the prime denotes $\partial/\partial u$. The fact that these equations are the same for both A_+ and A_- means that the equations for both Θ_+ and Θ_- can be simultaneously diagonalized by the same matrix S.

We notice that we have two differential equations for four unknown functions, which simply signifies that given a transformation matrix S which diagonalizes (3.17) then the same will be true if we multiply S by an arbitrary diagonal matrix. The final result of our calculation must of course be independent of these two extra degrees of freedom, so we may choose to fix them by imposing the convenient constraints

$$\begin{aligned} a' + b\tanh u &= 0\,, \\ c' + d\tanh u &= 0\,. \end{aligned} \tag{3.22}$$

This choice considerably simplifies S and $H_\pm$, which we find now takes the form

$$H_\pm = \begin{pmatrix} i(\omega \mp \frac{G}{2}) & 0 \\ 0 & i(\omega \mp \frac{G}{2}) \end{pmatrix}, \tag{3.23}$$

completely independent of the transformation matrix elements a, b, c, d. Even more remarkably, the resulting diagonal equation $(\partial_u - H_\pm)\Theta'_\pm = 0$ is solved by an arbitrary constant spinor times $e^{i\omega u} e^{\pm i\chi}$, where χ is precisely the same function which appeared in the zero mode solutions of [17] (and shown in (3.38) below).

All that remains is to determine the elements of the transformation matrix S. With the choice (3.22), (3.21) becomes

$$\begin{aligned} b' + a\tanh u - 2i\omega b &= 0\,, \\ d' + c\tanh u - 2i\omega d &= 0\,. \end{aligned} \tag{3.24}$$

The general solution to the coupled system (3.22), (3.24) is

$$S = \begin{pmatrix} a(u) & b(u) \\ c(u) & d(u) \end{pmatrix} = S_0 \begin{pmatrix} -|\omega + k|\, a_1(u) & e^{2i\omega u} b_1(u) \\ -|\omega - k|\, a_2(u) & e^{2i\omega u} b_2(u) \end{pmatrix}, \tag{3.25}$$

where S_0 is an arbitrary constant matrix (though we want it to be invertible),

$$\begin{aligned} b_1(u) &= e^{i(-\omega-k)u} e^{\frac{i}{2}\left(\tan^{-1}(\omega \sinh 2u)+\tan^{-1}(k \tanh 2u)\right)} \operatorname{sech} u \sqrt{|\omega \cosh 2u - k|}\,, \\ b_2(u) &= e^{i(-\omega+k)u} e^{\frac{i}{2}\left(\tan^{-1}(\omega \sinh 2u)-\tan^{-1}(k \tanh 2u)\right)} \operatorname{sech} u \sqrt{|\omega \cosh 2u + k|}\,, \end{aligned} \tag{3.26}$$

the $a_i(u)$ are obtained from the $b_i(u)$ by replacing $\omega \to -\omega$, and we have introduced

$$k = +\sqrt{\omega^2 - 1}. \tag{3.27}$$

We restrict our attention to $|\omega| \geq 1$ so that k is real and we get plane wave normalizable solutions.

Next we transform back to our initial spinors $\Theta_\pm = S^{-1}\Theta'_\pm$. We find that the inverse of S is simply

$$\Theta_\pm = S^{-1}\Theta'_\pm = -\frac{1}{4\omega k} \begin{pmatrix} b_2(u) & -b_1(u) \\ e^{-2i\omega u}|\omega - k|\, a_2(u) & -e^{-2i\omega u}|\omega + k|\, a_1(u) \end{pmatrix} S_0^{-1}\Theta'_\pm. \tag{3.28}$$

Recalling that $\Theta'_\pm$ was found to be given by $e^{i\omega u}e^{\pm i\chi}$ times an arbitrary constant spinor it is clear that we can absorb the constants of integration S_0^{-1} into the choice of constant spinor. Then the most general solution for $\vartheta^1_\pm$ (the top component of $\Theta_\pm$) is of the form

$$\vartheta^1_\pm = e^{i\omega u} e^{\pm i\chi} \left[b_2(u) U^2_\pm + b_1(u) U^1_\pm \right], \tag{3.29}$$

where $U^i_\pm$ are arbitrary constant spinors satisfying $\Gamma_{\phi\theta} U^i_\pm = \pm i U^i_\pm$.

However it is clear from (3.26) that $b_1(u)$ and $b_2(u)$ differ only by $k \leftrightarrow -k$. Therefore it is more transparent to treat the wavenumber k, rather than the frequency ω, as the parameter of the solution. Restoring the $e^{-i\omega v}$ dependence from (3.15), we therefore conclude that the most general solution to (3.12) is

$$\vartheta^1 = \operatorname{sech} u \sqrt{|\omega \cosh 2u + k|}\, e^{i\alpha} \left[e^{+i\chi} U_+ + e^{-i\chi} U_- \right], \tag{3.30}$$

where we now allow $k \in (-\infty, +\infty)$ with $\omega^2 = k^2 + 1$, $U_\pm$ are arbitrary (k-dependent) Weyl spinors satisfying $\Gamma_{\phi\theta} U_\pm = \pm i U_\pm$, and

$$e^{i\alpha} = e^{-i\omega v + iku} e^{\frac{i}{2}\left(+\tan^{-1}(\omega \sinh 2u) - \tan^{-1}(k \tanh 2u)\right)}. \tag{3.31}$$

Far away from the core of the magnon ($|u| \gg 1$) the solution behaves like $e^{-i\omega v + iku}$, representing a free fermion of mass $m^2 = 1$.

Of course the second order equation (3.5) admits both positive frequency ($\omega > 0$) and negative frequency ($\omega < 0$) solutions, and we have been careful in writing (3.30) to allow both cases. However the two kinds of solutions are clearly related to each other by complex conjugation (together with relabeling $k \to -k$ and $U_\pm \to U^*_\mp$) so we do not need to treat them separately. In what follows we focus on the positive frequency solutions, postponing discussion of the Majorana condition $(\Psi^I)^* = \Psi^I$ until the end.

3.3 The κ-fixed solutions

It remains to impose the κ symmetry projection (3.10). After some elementary Dirac matrix algebra we find the positive frequency mode

$$\Psi^1 = i\,\mathrm{sech}\, u\sqrt{\omega\cosh 2u + k}\, e^{i\alpha}\left[e^{+i\chi}\Gamma_0 + e^{-i\chi}\Gamma_\phi\right](U_+ - \Gamma_0\Gamma_\phi U_-)\,. \tag{3.32}$$

Let us now address the counting of degrees of freedom. Before imposing any chirality condition, each $U_\pm$ has 16 (complex) components, so it might appear that (3.32) involves 32 independent components. However if we assemble $U_\pm$ into a single 32-component spinor $U = U_+ + U_-$ according to

$$U_\pm = \frac{1}{2i}(i \pm \Gamma_{\phi\theta})U\,, \tag{3.33}$$

then the combination appearing in (3.32) is

$$(U_+ - \Gamma_0\Gamma_\phi U_-) = \mathcal{P}U\,, \qquad \mathcal{P} = \frac{1}{2i}\left[(i+\Gamma_{\phi\theta}) - \Gamma_0\Gamma_\phi(i-\Gamma_{\phi\theta})\right]. \tag{3.34}$$

It is not hard to check that $\mathcal{P}$ is a projection matrix with half maximal rank, so in fact (3.32) only involves the 16 components of U which are not killed by $\mathcal{P}$. Moreover $\mathcal{P}$ commutes with the chirality operator Γ^{11} and has eight nontrivial eigenspinors of each chirality. Therefore it is consistent to impose a Weyl condition on U further reducing the number of components by half to just eight.

To summarize, we find the general κ-fixed positive frequency modes

$$\Psi^1 = i\sqrt{\omega+k}\,\csc\tfrac{p}{4}\,\mathrm{sech}\, u\sqrt{\omega\cosh 2u + k}\, e^{i\alpha}\left[e^{+i\chi}\Gamma_0 + e^{-i\chi}\Gamma_\phi\right]\mathcal{P}U\,, \tag{3.35}$$

where we have introduced a convenient but otherwise arbitrary overall factor, U is an arbitrary (k-dependent) complex Weyl spinor, $\mathcal{P}$ is the projection operator defined in (3.34) and

$$\omega = +\sqrt{k^2+1}. \tag{3.36}$$

Then from (3.11) we find that the second spinor is determined to be

$$\Psi^2 = \sqrt{\omega-k}\,\sec\tfrac{p}{4}\,\mathrm{sech}\, u\sqrt{\omega\cosh 2u - k}\, e^{i\beta}\,\Gamma_*\Gamma_\phi\left[e^{+i\tilde\chi}\Gamma_0 + e^{-i\tilde\chi}\Gamma_\phi\right]\mathcal{P}U\,. \tag{3.37}$$

In these expressions χ and $\tilde\chi$ are as defined in [17],

$$\begin{aligned} e^{i\chi} &= \left(\frac{\sinh u + i\cos\frac{p}{2}}{\sinh u - i\cos\frac{p}{2}}\right)^{1/4}(\tanh u + i\,\mathrm{sech}\, u)^{1/2} \\ e^{i\tilde\chi} &= \left(\frac{\sinh u - i\cos\frac{p}{2}}{\sinh u + i\cos\frac{p}{2}}\right)^{1/4}(\tanh u + i\,\mathrm{sech}\, u)^{1/2}\,, \end{aligned} \tag{3.38}$$

while α and β are given by

$$\begin{aligned} e^{i\alpha} &= e^{-i\omega v + iku}e^{\frac{i}{2}\left(+\tan^{-1}(\omega\sinh 2u) - \tan^{-1}(k\tanh 2u)\right)} \\ &= e^{-i\omega v + iku}\left(\frac{1+i\omega\sinh 2u}{1-i\omega\sinh 2u}\,\frac{1-ik\tanh 2u}{1+ik\tanh 2u}\right)^{1/4}, \\ e^{i\beta} &= e^{-i\omega v + iku}e^{\frac{i}{2}\left(-\tan^{-1}(\omega\sinh 2u) - \tan^{-1}(k\tanh 2u)\right)} \\ &= e^{-i\omega v + iku}\left(\frac{1-i\omega\sinh 2u}{1+i\omega\sinh 2u}\,\frac{1-ik\tanh 2u}{1+ik\tanh 2u}\right)^{1/4}. \end{aligned} \tag{3.39}$$

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Finally we come to the Majorana condition $(\Psi^I)^* = \Psi^I$. There are 16 complex linearly independent solutions to the equations of motion (3.5): eight positive frequency solutions shown in (3.35) and (3.37) and eight corresponding negative frequency solutions given by their complex conjugates. After imposing the Majorana condition only eight complex linearly independent solutions remain. Upon quantization the coefficient U becomes an operator and the Majorana condition relates the coefficient of positive frequency modes to the adjoint of the coefficient of negative frequency modes, resulting in a total spectrum of eight different kinds of fermionic particles.

It is also easy to exhibit manifestly real Majorana solutions by starting with the particular linear combination

$$\Psi^1 = i\sqrt{\omega+k}\,\csc\tfrac{p}{4}\,\operatorname{sech}u\sqrt{\omega\cosh 2u+k}\sum_{\pm}(e^{\pm i\chi}\Gamma_0+e^{\mp i\chi}\Gamma_\phi)(e^{+i\alpha}U^1_\pm+e^{-i\alpha}U^2_\pm) \quad (3.40)$$

of positive and negative frequency modes. In a Majorana representation of the Γ matrices, in which $(\Gamma^\mu)^* = -\Gamma^\mu$, Γ^0 being antisymmetric and the rest symmetric, imposing the Majorana condition $(\Psi^1)^* = \Psi^1$ on (3.40) amounts to imposing

$$(U^1_+)^* = U^2_-\,, \qquad (U^2_+)^* = U^1_-\,. \quad (3.41)$$

It is then not hard to see that Ψ^1 depends only on the real parts of U^1_+ and U^2_+, which we will call $U^{1,2}$ respectively. We conclude by displaying the resulting manifestly real solutions

$$\begin{aligned}\Psi^1 &= 2i\sqrt{\omega+k}\,\csc\tfrac{p}{4}\,\operatorname{sech}u\sqrt{\omega\cosh 2u+k}\\ &\quad\times[\Gamma_0(\cos\chi+\sin\chi\,\Gamma_{\phi\theta})+\Gamma_\phi(\cos\chi-\sin\chi\,\Gamma_{\phi\theta})]\\ &\quad\times\left[(\cos\alpha+\sin\alpha\,\Gamma_{\phi\theta})U^1+(\cos\alpha-\sin\alpha\,\Gamma_{\phi\theta})U^2\right],\end{aligned} \quad (3.42)$$

and

$$\begin{aligned}\Psi^2 &= 2\sqrt{\omega-k}\,\sec\tfrac{p}{4}\,\operatorname{sech}u\sqrt{\omega\cosh 2u-k}\,\Gamma_*\Gamma_\phi\\ &\quad\times[\Gamma_0(\cos\tilde\chi+\sin\tilde\chi\,\Gamma_{\phi\theta})+\Gamma_\phi(\cos\tilde\chi-\sin\tilde\chi\,\Gamma_{\phi\theta})]\\ &\quad\times\left[(\cos\beta+\sin\beta\Gamma_{\phi\theta})U^1+(\cos\beta-\sin\beta\Gamma_{\phi\theta})U^2\right],\end{aligned} \quad (3.43)$$

which have been written in a form similar to (2.25) and (2.28) of [17].

4. The 1-loop functional determinant

We now apply the results of the previous two sections to check the leading quantum correction to the energy of the giant magnon, following well-established techniques for the semiclassical quantization of solitons (see in particular [46, 4, 3], and [53] for a review). Of course if energy is defined as the Noether charge of the actions (2.1) and (3.1) associated with worldsheet t translations then the energy of any physical string state is identically zero due to the Virasoro constraints. Rather than this kind of energy we are instead interested in the conserved quantity

$$\Delta - J = \frac{\sqrt{\lambda}}{2\pi}\int_{-\infty}^{+\infty} dx\,\left[(Y^0\partial_t Y^5 - Y^5\partial_t Y^0) - (X^5\partial_t X^6 - X^6\partial_t X^5)\right], \quad (4.1)$$

where Δ is the charge associated with global time translations in AdS_5 (i.e., the spacetime energy) and J is the U(1) charge associated with rotations in the equator of the S^5. The quantity $\Delta - J$ can be identified with the hamiltonian of physical string excitations [44]. In light-cone gauge this would be the familiar transverse hamiltonian. In conformal gauge we have to supplement the sigma-model action with ghosts to cancel two unphysical bosonic degrees of freedom. As discussed in section 2, it is sufficient for the purpose of our semi-classical analysis to instead simply omit the two massless bosonic modes we found above.

The classical value of $\Delta - J$ for the giant magnon solution (2.6) is

$$\Delta - J = \frac{\sqrt{\lambda}}{\pi}\left|\sin\frac{p}{2}\right|. \tag{4.2}$$

The one-loop semiclassical correction to this result comes from evaluating the functional determinant $\ln\det|\delta^2 S|$ around the classical configuration. The reader is surely familiar with the fact that the contribution from this determinant is

$$\frac{1}{2}\sum_k(-1)^F\nu_k \tag{4.3}$$

where $(-1)^F$ is $+1$ for bosons and -1 for fermions, and ν_k are the frequencies of small oscillations around the classical background. This is manifestly independent of $\sqrt{\lambda}$ and hence constitutes the $\mathcal{O}(1/\sqrt{\lambda})$ correction to the classical result (4.2).

For a soliton at rest the ν_n are simply given by the masses of the fluctuations. The giant magnon moves with velocity

$$\mathrm{v} = \cos\tfrac{p}{2} \tag{4.4}$$

and is only at rest for the special case $p = \pi$. The calculation of (4.3) in this case was performed in [33], where it followed as a special case of a study of various more general spinning string configurations. In fact the authors of [33] found that the first $\mathcal{O}(1/\sqrt{\lambda})$ correction to (4.2) vanishes, in agreement with the expectation based on supersymmetry [15, 5] that the exact result for $p = \pi$ should be

$$\Delta - J = \sqrt{1+\frac{\lambda}{\pi^2}} = \frac{\sqrt{\lambda}}{2}\left[1 + \frac{0}{\sqrt{\lambda}} + \mathcal{O}(1/\lambda)\right]. \tag{4.5}$$

For general p the expected relation [15, 5] is

$$\Delta - J = \sqrt{1+\frac{\lambda}{\pi^2}\sin^2\frac{p}{2}}, \tag{4.6}$$

which also has a vanishing $\mathcal{O}(1/\sqrt{\lambda})$ correction to the classical value (4.2). Now for general p the magnon is no longer static so we employ the method of Dashen, Hasslacher and Neveu [46] to calculate the so-called stability angles ν_n which enter in the one-loop functional determinant (4.3). We begin by putting the system in a box of length $L \gg 1$, identifying $x \cong x + L$. From (2.6) it is clear that the system is also periodic in time with period $T = L/\mathrm{v}$. The stability angle ν of a generic fluctuation $\delta\phi$ is defined as

$$\delta\phi(t+T, x) = e^{-i\nu}\delta\phi(t,x). \tag{4.7}$$

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It is straightforward to read off the stability angles for the fluctuations around the giant magnon from our results in sections 2 and 3. The four free massive AdS_5 bosons behave as $e^{iku-i\omega v}$ and hence trivially have

$$\nu_k(\delta Y) = \frac{L}{\mathrm{v}} \frac{\omega + \mathrm{v}k}{\sqrt{1-\mathrm{v}^2}} \,. \tag{4.8}$$

For the four physical S^5 fluctuations we find from (2.20) that

$$\nu_k(\delta X) = \frac{L}{\mathrm{v}} \frac{\omega + \mathrm{v}k}{\sqrt{1-\mathrm{v}^2}} + 2\cot^{-1} k \,. \tag{4.9}$$

Finally from the explicit form of the eight fermions in section 4 we find

$$\nu_k(\vartheta) = \frac{L}{\mathrm{v}} \frac{\omega + \mathrm{v}k}{\sqrt{1-\mathrm{v}^2}} + \cot^{-1} k \,. \tag{4.10}$$

Summing these results, with a minus sign for the fermions, gives precisely the expected result that the one-loop correction to (4.2) vanishes (even before integrating over k), thereby providing a check on our results.

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